MODAL ANALYSIS OF THE SLOTTED-CIRCULAR COAXIAL CAVITIES USED IN SPACE-HARMONIC MILLIMETER WAVE MAGNETRONS

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ABSTRACT

Computer-aided design of millimeter wave magnetrons operating in a non-π mode calls for self-consistent numerical simulation of the electron dynamics. For a modal analysis of the slotted-circular coaxial cavities used in these magnetrons, complete sets of eigenmodes, resonant as well as irrotational, have to be computed. The Generalized Spectral Domain (GSD) method provides a fast and accurate mean to calculate the eigenvalues of these modes and to investigate their modal field distributions. Results for a typical millimeter wave magnetron cavity are presented.

INTRODUCTION

One of the technological difficulties met when designing compact and reliable classical magnetrons at frequencies higher than 90 GHz are related to the high magnetic field required to operate in the fundamental π mode. A radical solution to this problem consists in adopting one of the $\pi/2$ mode as the operating mode [1]. Interaction of the electrons with the electromagnetic fields then occurs with the first backward space harmonic of this mode. Although this new design feature led to improvements with regard to output power, adequate description of the electron dynamics in such type of magnetron does not exist at present.

To simulate the interaction between the electrons and the generated electromagnetic fields, accurate computation of the modal field distributions of the magnetron structure is required. The modal analysis of the cavity results from the projection of the electromagnetic field on the complete set of eigenfunctions, resonant as well as irrotational modes. Irrotational electric eigenmodes account partly for the dynamic and static space charge effects. Depending on the axial boundary conditions chosen in their definitions (short- or open-circuit condition), irrotational magnetic modes account, also partly, for the coupling apertures.

The magnetron structure analysed in this contribution is a lossless coaxial cavity of uniform cross-section with slots cut in the anode block which are periodically spaced in azimuthal direction. A mean to calculate the modal field distributions of such a composite structure consists in applying the Generalized Spectral Domain (GSD) method [2] to a section of the cavity. The three-dimensional eigenvalue problems for the resonant and irrotational modes are then solved by considering the axial boundary conditions.

THEORY

The GSD method applied to a section of the azimuthally periodic circular coaxial cavity of length L coupled to N sectors leads to the determination of the cutoff wavenumbers of the TE and TM eigenmodes and of their associated field distributions. The method is based on short-circuiting one coupling surface and replacing the tangential electric field at its boundary by two surface magnetic currents on both sides of the short-circuit. Imposing that the two surface magnetic currents, which are to be computed, are equal in magnitude but opposite in direction makes the tangential electric field continuous across the boundary. The azimuthal periodicity is taken into account by applying the Floquet's theorem. The GSD method needs then to be applied only to one single coupling surface.

The projection of the electromagnetic field on the eigenmodes of the coaxial and sector waveguides leads to relations between the expansion coefficients and the

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unknown surface magnetic current in the form of integral equations. For the determination of the TE (resp. TM) modes of the composite waveguide, these equations involve the axial (resp. azimuthal) component of the surface current. Expanding the axial component of the surface current on the sets of functions:

$$\frac{\cos(\frac{k\pi}{\theta}\phi)}{\sqrt[3]{\left(\frac{\theta}{2}\right)^2-\phi^2}}, \qquad k=0,2,4...,$$

and:

$$\frac{\sin(\frac{k\pi}{\theta}\phi)}{\sqrt[3]{\left(\frac{\theta}{2}\right)^2-\phi^2}}, \qquad k=1,3,5...,$$

and the azimuthal component on the sets:

$$\frac{\cos(\frac{k\pi}{\theta}\phi)}{\sqrt[3]{\left(\frac{\theta}{2}\right)^2-\phi^2}}, \qquad k=1,3,5...,$$

and:

$$\frac{\sin(\frac{k\pi}{\theta}\phi)}{\sqrt[3]{\left(\frac{\theta}{2}\right)^2-\phi^2}}, \qquad k=2,4,6...,$$

then, testing the continuity of the axial and azimuthal magnetic field (Galerkin's procedure) across the coupling surface on the same sets results in the TE and TM characteristic systems of equations, respectively, of the type:

$$[Y^w+Y^s]\mathbf{V}=0.$$

In the above relation, $\mathbf{V}$ is a column voltage vector the elements of which are the expansion coefficients of the relevant components of the surface magnetic current. $[Y^w]$ and $[Y^s]$ are admittance matrices associated with the coaxial and the sector waveguides regions, the terms of which are functions of the searched cutoff wavenumbers k_c of the composite waveguide. The TM-matrix equation is determined by considering the TE, TM and TEM eigenmodes of the coaxial waveguide and the TE and TM eigenmodes of the sector waveguide. Only the TE eigenmodes of the coaxial and sector waveguides have to be taken into consideration to obtain the TE-admittances matrices. Note that the basis functions satisfy the edge condition for an edge angle of $\pi/2$. This requirement makes the procedure numerically very efficient [2].

The voltage eigenvectors $\mathbf{V}$ determines the potential functions $\phi^h_{nm}(r,\theta)$ and $\phi^e_{nm}(r,\theta)$ of the composite waveguide, with cutoff wavenumbers $k^h_{c,nm}$ and $k^e_{c,nm}$, from which the field distributions of the TE and TM eigenmodes are derived. For a given index n associated with phase shift per sector $2\pi n/N$, $k^i_{c,nm}$ (i corresponds to h or e) is the m^{th} cutoff wavenumber which singularizes the corresponding matrix equation.

The TE$_{nml}$ and TM$_{nml}$ eigenfrequencies of the cavity are readily obtained by imposing axial boundary conditions of the short- or open-circuit type. Regarding the irrotational eigenmodes [3], their their field distributions are derived from the potential functions $\Phi_{nml}(r,\theta,z) = \phi^e_{nm}(r,\theta)f_l(z)$ and $\Psi_{nml}(r,\theta,z) = \phi^h_{nm}(r,\theta)g_l(z)$. Φ_{nml} and Ψ_{nml} are associated with the irrotational electric $\mathbf{F}_{nml}$ and magnetic $\mathbf{G}_{nml}$ eigenfunctions, respectively. Functions $f_l(z)$ and $g_l(z)$ are the $\cos k_{z,l}z$ or $\sin k_{z,l}z$ functions, where $k_{z,l} = l\pi/L$ (l being integer), depending on the imposed axial boundary conditions. For each type of these (short-circuit or open-circuit), the eigenvalues associated to the respective irrotational electric eigenfunctions are given by:

$$p^2_{nml} = \left(k^e_{c,nm}\right)^2 + k^2_{z,l}.$$

The eigenvalues for the two types of irrotational magnetic eigenfunctions are given by:

$$q^2_{nml} = \left(k^h_{c,nm}\right)^2 + k^2_{z,l}.$$

RESULTS

The computer code which has been developed for the computation of the resonant and irrotational modes has been used to perform a complete modal analysis of a 94 GHz magnetron cavity. This cavity has $N = 28$ slots. Each slot has an opening of 8.5^o. The ratio of the slot depth to the anode radius h/r_a is 0.46. The ratio of the cathode radius to the anode one is 0.54.

Fig. 1 shows the resonant frequencies of the TE$_{n10}$ and TE$_{n20}$ eigenmodes ($n = 0,1,2,...N/2$) with axial open-boundary conditions plotted in a dispersion diagram representation. Points in the two passbands (denoted by the solid lines) corresponding to phase shifts between 0 and π and between π and 2π are associated with the fundamental and the first backward space harmonic, respectively. Fig. 2 shows the electric field lines of the TE_{710} mode ($\pi/2$ mode). Fig. 3 and Fig. 4 illustrate, for $n = 6$, the second-order degeneracy of the modes $n \neq 0$ and $n \neq N/2$.

Fig. 5 shows the influence of the variation of the slot depth on the frequencies of the first passband. Decreasing the slot depth always leads to an increase of these eigenfrequencies. The effect becomes more remarkable when the mode number n is higher. The influence of the number of slots on the frequencies of the TE_{n10} modes is depicted on Fig. 6 for $n = N/4$, $n = N/4 - 1$ and $n = N/4 + 1$. It is seen that, keeping the mode number n constant, an increase of the number of slots does not affect these eigenfrequencies very significantly. On the other hand, the eigenfrequencies of the $\pi/2$ mode always increase with the number of slots.

As an example of computation of irrotational eigenfunctions, in Fig. 7, the contour lines of the potential function associated with one of the eigenmodes $\mathbf{F}_{610}$ is presented. Note that these contour lines correspond also to the magnetic field lines of one of the TM_{610} mode.

CONCLUSIONS

A method has been developed to perform a complete modal analysis of millimeter wave magnetron structures. This method uses the GSD technique to find the eigenfrequencies and eigenvalues of the resonant and irrotational modes. Numerical results have been presented for a 94 GHz magnetron structure. The method used in this contribution paves the way for the self-consistent numerical simulations and the optimization of modern high-power miniaturized magnetrons at frequencies as high as 230 GHz.

ACKNOWLEDGEMENTS

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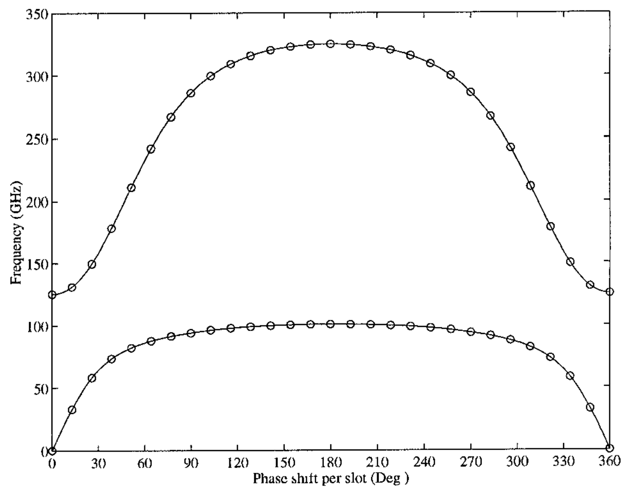

Fig. 1: Dispersion diagram associated with the TE_{n10} and TE_{n20} modes

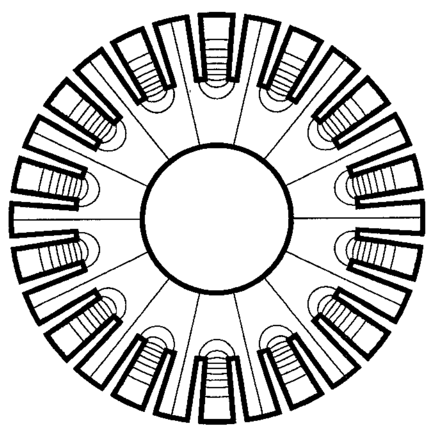

Fig. 2: Electric field lines of the TE_{710} mode

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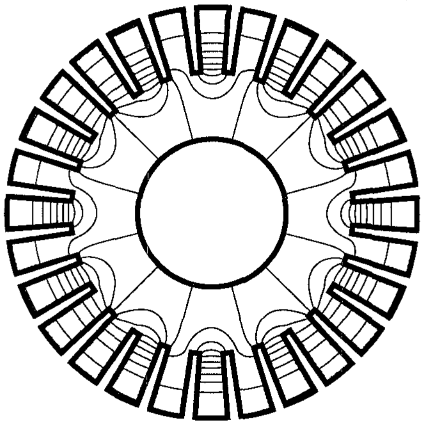

Fig. 3: Electric field lines of the TE_{610} mode (even angularly dependence)

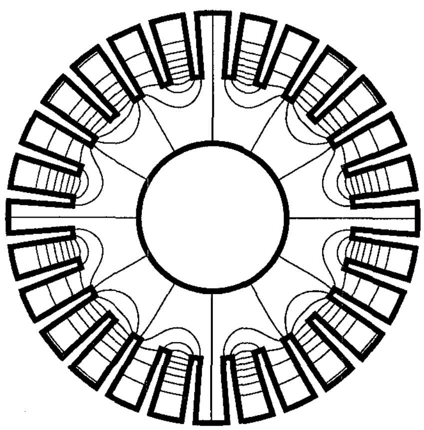

Fig. 4: Electric field lines of the TE_{610} mode (odd angularly dependence)

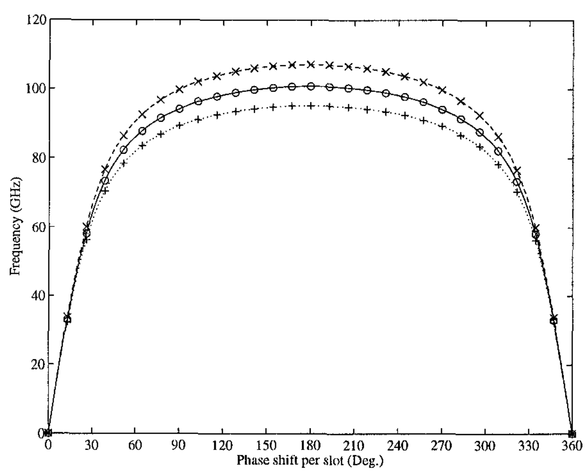

Fig. 5: Variation of the TE_{n10} frequencies with the ratio h/r_a; (o): $h/r_a = 0.46$, (x): $h/r_a - 2\%$, (+): $h/r_a + 2\%$

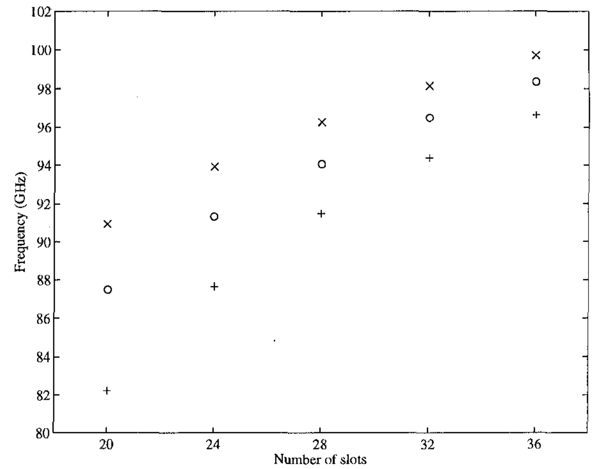

Fig. 6: Variation of the TE_{n10} frequencies with the number of slots N; (o): $n = N/4$ ($\pi/2$ modes), (+): $n = N/4 - 1$, (x): $n = N/4 + 1$

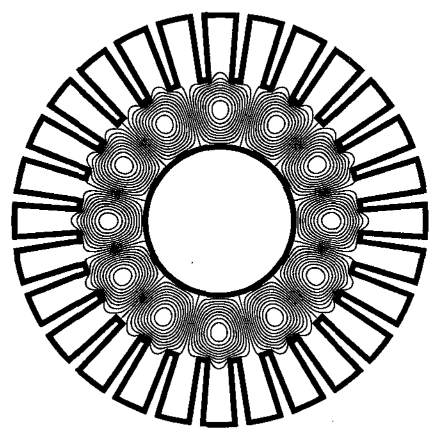

Fig. 7: Contour lines of the potential that correspond to the $\mathbf{F}_{610}$ mode (even angularly dependence)